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Forced Convection of Power-Law Fluids Flow over a Rotating Nonisothermal Body

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Presented is an analysis of steady laminar flow of power-law fluids past a rotating body with nonisothermal surfaces. A coordinate transformation combined with the Merk-type series expansion is employed to transform the governing momentum equations into a set of coupled ordinary differential equations. The equations are numerically integrated to obtain the axial and tangential velocity gradients for determining the friction coefficient. For forced convection, a generalized coordinate transformation is used to analyze the temperature field of the power-law flow. Solutions to the transformed energy equations are obtained in the form of universal functions. The heat transfer coefficients in terms of $NuRe^{1/(n+1)}$ are presented for a rotating sphere. The effects of power-law index, rotation parameter, Prandtl number, and the location of step discontinuity in surface temperature on the local Nusselt number are fully investigated and demonstrated.

Nomenclature

$a_m(\xi)$ = coefficients defined in Eq. (28)
$b(\xi)$ = coefficient defined in Eq. (30)
b_i = geometry coefficient
C_f = friction coefficient
c = coefficient defined in Eq. (30)
c_i = geometry coefficient
d_i = flow coefficient
f = dimensionless stream function
g = dimensionless tangential velocity
$H(x - x_0)$ = Heaviside function
K = coefficient in apparent viscosity
k = thermal conductivity
L = characteristic length
M = coefficient in temperature function
N = coefficient in temperature function
Nu = Nusselt number
n = fluid power-law index
P = coefficient in temperature function
Pr = Prandtl number
$p_k(\xi)$ = coefficients defined in Eq. (29)
Q = coefficient in temperature function
q_w = wall heat flux
R = radius of a rotating sphere
Re = generalized Reynolds number, $\rho L^n(U_\infty)^{2-n}/K$
r = radius of an axisymmetric body
T_s = surface temperature of axisymmetric body starting at x_0
T_∞ = temperature of incoming fluid
U_e = velocity at the outer edge of the boundary layer
U_∞ = velocity of incoming fluid
u = velocity in x direction
v = velocity in y direction
W = rotation parameter, $L\Omega/U_\infty$
w = tangential velocity in direction of rotation
X = transformed dimensionless coordinate defined in Eq. (21a)
x = coordinate measured along surface from forward stagnation point
x_0 = location where body temperature has a discontinuity
y = coordinate normal to surface
α = thermal diffusivity
β = parameter defined in Eq. (23)
$\Gamma(n, \zeta^3)$ = incomplete gamma function defined in Eq. (41)
ζ = transformed dimensionless coordinate defined in Eq. (21b)
η = transformed dimensionless coordinate defined in Eq. (6b)
θ = dimensionless temperature defined in Eq. (21c)
Λ = wedge parameter
ξ = transformed dimensionless coordinate defined in Eq. (6a)
ξ_0 = transformed dimensionless coordinate of x_0
ρ = fluid density
τ = shear stress
ψ = transformed dimensionless coordinate defined in Eq. (6c)
Ω = angular velocity

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Introduction

THE flow characteristics created by a rotating body placed in a uniform fluid stream and the subsequent effect on heat transfer has been of interest to many investigators due to the frequent use of rotating machinery in industrial processes. Some of these processes require non-Newtonian fluids that are characterized by the power-law model. The transport analysis of power-law flow over a rotating body is inherently more complicated by the additional nonlinearity caused by the power-law index. This fact significantly reduces the possibility of successful applications of similarity solutions and other transformation techniques that have been used to analyze rotating bodies in Newtonian fluid flows.

Forced convection over a rotating disk in Newtonian flow was examined by Sparrow and Gregg,[1] Hartnett and Deland,[2] and Tien and Tsuji.[3] Chao and Greif[4] considered laminar forced convection over rotating bodies with or without uniform stream. A computational procedure was applied to a rotating sphere and a rotating disk with nonuniform surface temperature by using a two-term velocity profile and a coordinate transformation for the energy equation. Lee et al.[5] investigated momentum and heat transfer over a rotating body in forced flow by employing the Merk-type series expansion. Numerical examples of a rotating disk and a sphere were presented. Forced convection over a rotating body with nonuniform surface temperature was analyzed by Jeng et al.[6] They used a unique coordinate transformation that was first proposed by Chao and Cheema[7] in considering forced convection in wedge flow with nonisothermal surfaces. All of the above-mentioned work was related to Newtonian fluids.

During the last two decades, several investigators attempted to employ coordinate transformations combined with an asymptotic series expansion in dealing with the power-law flow over a variety of stationary bodies. Chen and Radulovic[8] analyzed the heat transfer in non-Newtonian flow past a wedge with nonisothermal surfaces. Kim et al.[9] examined the characteristics of boundary-layer transfer in power-law flow over a two-dimensional or axisymmetric body with nonisothermal surfaces. Kleintreuer and Wang[10] presented a numerical analysis of the mixed thermal convection of power-law fluids past standard bodies with suction or injection and axisymmetric body rotation. An implicit finite-difference scheme was employed to solve the governing equations which were simplified by a coordinate transformation. Recently, a similarity solution was reported by Wang and Kleinstreuer[11] for combined convection heat transfer from a rotating cone or disk to non-Newtonian fluids.

This article considers the momentum and heat transfer in power-law flows over a rotating body with nonisothermal surfaces. A generalized coordinate transformation is used to transform the governing equation of motion. The Merk-type series expansion is employed to represent both the axial velocity and the tangential velocity created by the body spin. Then the energy equation is transformed into a sequence of second-order linear differential equations. The solution to those equations is expressed in terms of universal functions that are independent of the body geometry. The local friction and heat transfer coefficients for a rotating sphere are presented for an illustration.

Problem Description and Solution Method

The problem considered in this article is a steady, incompressible, laminar boundary layer over a rotating axisymmetric body placed in a uniform stream of power-law fluids. The body is spinning at a constant Ω with its axis parallel to the direction of the freestream. The front portion of the object measured from the forward stagnation point to an arbitrary distance x_0 is isothermal and possesses the freestream temperature of T_∞. At the location x_0 the wall temperature is step-changed to T_s, as shown in Fig. 1. It is assumed that momentum and thermal properties are constant without external body forces and viscous dissipation.

The governing boundary-layer equations, upon imposing the above-stated conditions, are

Continuity

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{r}\frac{\mathrm{d}r}{\mathrm{d}x} = U_e\frac{\mathrm{d}U_e}{\mathrm{d}x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{2a}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \frac{uw}{r}\frac{\mathrm{d}r}{\mathrm{d}x} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(K\left|\frac{\partial w}{\partial y}\right|^{n-1}\frac{\partial w}{\partial y}\right) \tag{2b}$$

with boundary conditions

$$@\, y = 0 \quad u = v = 0, \quad w = r\Omega \tag{3a}$$

$$\text{for } y \to \infty \quad u = U_e, \quad v = w = 0 \tag{3b}$$

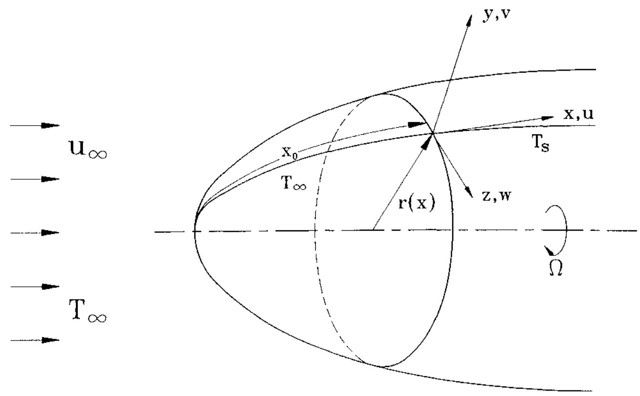

Fig. 1 Physical model and the coordinate system.

Energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

with the boundary conditions

$$T(x, 0) = T_\infty + (T_s - T_\infty)H(x - x_0) \tag{5a}$$

$$T(x, \infty) = T_\infty \tag{5b}$$

The momentum equations are solved first by using a coordinate transformation with new dimensionless variables, stream function, and tangential velocity component defined as follows:

$$\xi = n\int_0^x \left(\frac{r}{L}\right)^{n+1}\left(\frac{U_e}{U_\infty}\right)^{2n-1}\frac{\mathrm{d}x}{L} \tag{6a}$$

$$\eta = \left[\frac{Re}{(n+1)\xi}\right]^{1/(n+1)}\left(\frac{U_e}{U_\infty}\right)\left(\frac{r}{L}\right)\frac{y}{L} \tag{6b}$$

$$\psi = \left[\frac{(n+1)\xi}{Re}\right]^{1/(n+1)}(U_\infty L^2)f(\xi, \eta) \tag{6c}$$

$$w = r\Omega g(\xi, \eta) \tag{6d}$$

With the stream function defined by

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \qquad v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$$

the momentum equations are now transformed to

$$(|f''|^{n-1}f'')' + nff'' + n\Lambda[1 - (f')^2] + \frac{n(n+1)\xi}{r}\frac{\mathrm{d}r}{\mathrm{d}\xi} \times \left(\frac{r\Omega}{U_e}\right)^2 g^2 = n(n+1)\xi\left(f'\frac{\partial^2 f}{\partial\xi\partial\eta} - \frac{\partial f}{\partial\xi}f''\right) \tag{7a}$$

$$(|g'|^{n-1}g')' + n\left(\frac{r\Omega}{U_e}\right)^{1-n}fg' - \frac{2n(n+1)\xi}{r}\frac{\mathrm{d}r}{\mathrm{d}\xi}f'g = n(n+1)\xi\left(\frac{r\Omega}{U_e}\right)^{1-n}\left(\frac{\partial f}{\partial\eta}\frac{\partial g}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\eta}\right) \tag{7b}$$

where the prime denotes the differentiation with respect to η, and the parameter Λ is defined as

$$\Lambda = \frac{(n+1)\xi}{U_e}\frac{\mathrm{d}U_e}{\mathrm{d}\xi} \tag{8}$$

The associated boundary conditions are

$$f(0) = f'(0) = 0, \qquad f'(\infty) = 1 \tag{9a}$$

$$g(0) = 1, \qquad g(\infty) = 0 \tag{9b}$$

The momentum boundary-layer equations, though transformed into a simpler form of equations, are not ordinary differential equations. In addition, the equations are highly nonlinear due to n. In this article, the Merk-Meksyn method of asymptotic series expansion is adopted since the intention is to solve simple, ordinary differential equations than partial differential equations. The method utilizes the fact that one-to-one correspondence between Λ and ξ can lead to the decomposition of equations into a set of ordinary differential equations. Then, the equations are numerically integrated at the given values of parameter Λ so that elaborate numerical computation can be avoided. Since the method is well-established and a detailed account can be found in earlier references,[5,9] only a brief explanation will be presented.

Motivated by the original series expansion by Merk,[12] and later improved by Chao and Fagbenle[13] for a Newtonian flow case, f and g are expanded by

$$f = f_0 + (n+1)\xi\frac{d\Lambda}{d\xi}f_1 + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}f_2 + \left[(n+1)\xi\frac{d\Lambda}{d\xi}\right]^2 f_3 + \cdots \tag{10a}$$

$$g = g_0 + (n+1)\xi\frac{d\Lambda}{d\xi}g_1 + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}g_2 + \left[(n+1)\xi\frac{d\Lambda}{d\xi}\right]^2 g_3 + \cdots \tag{10b}$$

Since the quantities

$$\frac{(n+1)\xi}{r}\frac{dr}{d\xi}\left(\frac{r\Omega}{U_e}\right)^2, \quad \frac{2(n+1)\xi}{r}\frac{dr}{d\xi}, \quad \left(\frac{r\Omega}{U_e}\right)^{1-n}$$

in Eqs. (7a) and (7b) are functions of x only, they are expressed in terms of Λ as follows:

$$\frac{(n+1)\xi}{r}\frac{dr}{d\xi}\left(\frac{r\Omega}{U_e}\right)^2 = W^2\left[b_0\Lambda + (n+1)\xi\frac{d\Lambda}{d\xi}b_1 + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}b_2 + \cdots\right] \tag{11}$$

$$\frac{2(n+1)\xi}{r}\frac{dr}{d\xi} = c_0\Lambda + (n+1)\xi\frac{d\Lambda}{d\xi}c_1 + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}c_2 + \cdots \tag{12}$$

$$\left(\frac{r\Omega}{U_e}\right)^{1-n} = W^{1-n}\left[d_0 + (n+1)\xi\frac{d\Lambda}{d\xi}d_1 + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}d_2 + \cdots\right]^{1-n} \tag{13}$$

where b_i, c_i, and d_i are determined by the characteristics of flow and geometry.

Substitution of the expression of Eqs. (10a) and (10b) into Eqs. (7a) and (7b) yields the following set of coupled ordinary differential equations:

$$f_0'''(|f_0''|^{n-2}f_0'') + f_0f_0'' + \Lambda[1 - (f_0')^2 + b_0W^2g_0^2] = 0 \tag{14a}$$

$$g_0''(|g_0'|^{n-2}g_0') + (d_0W)^{1-n}f_0g_0' - c_0(d_0W)^{1-n}\Lambda f_0'g_0 = 0 \tag{14b}$$

with the associated boundary conditions shown as

$$f_0(0) = f_0'(0) = 0, \quad f_0'(\infty) = 1 \tag{15a}$$

$$g_0(0) = 1, \quad g_0(\infty) = 0 \tag{15b}$$

$$f_1'''(|f_0''|^{n-2}f_0'') + (n-1)|f_0''|^{n-2}f_1''f_0''' + f_0f_1'' + f_1f_0'' - 2\Lambda f_0'f_1' + W^2(b_1g_0^2 + 2b_0\Lambda g_0g_1) + (n+1)\cdot(f_1f''0 - f_1'f_0') = \frac{\partial(f_0', f_0)}{\partial(\Lambda, n)} \tag{16a}$$

$$g_1''(|g_0'|^{n-2}g_0') - (n-1)g_1'g_0''|g_0'|^{n-2} + W^{1-n}d_0^{1-n}\{(f_1g_0' + f_0g_1') + (n-1)(d_0)^{-1}d_1f_0g_0' - c_0\Lambda(f_0'g_1 + f_1'g_0) - f_0'g_0[c_1 + (n-1)(d_0)^{-1}c_0\, d_1\Lambda]\} = W^{1-n}(d_0)^{1-n} \times \left[\frac{dg_0}{d\Lambda}f_0' - \frac{df_0}{d\Lambda}g_0' + (n+1)(g_1f_0' - f_1g_0')\right] \tag{16b}$$

with

$$f_1(0) = f_1'(0) = f_1'(\infty) = g_1(0) = g_1(\infty) = 0 \tag{17}$$

$$f_2'''(|f_0''|^{n-2}f_0'') + (n-1)|f_0''|^{n-2}f_0'''f_2'' + f_0f_2'' + 2(\Lambda - n - 1)f_0'f_2' + (2n+3)f_0''f_2 + W^2(b_2g_0^2 + 2b_0\Lambda g_0g_2) = f_1'f_0' - f_1'f_0'' \tag{18a}$$

$$g_2''(|g_0'|^{n-2}g_0') - (n-1)|g_0'|^{n-2}g_0''g_2' + W^{1-n}d_0^{1-n} \times \{(f_0g_2' + f_2g_0') + (n-1)(d_0)^{-1}d_2f_0g_0' - c_0\Lambda(f_0'g_2 + f_2'g_0) - [(n-1)c_0(d_0)^{-1}d_2\Lambda + c_2]f_0'g_0\} = W^{1-n}(d_0)^{1-n}[g_1f_0' + 2(n+1)g_2f_0' - f_1g_0' - 2(n+1)f_2g_0'] \tag{18b}$$

with

$$f_2(0) = f_2'(0) = f_2'(\infty) = g_2(0) = g_2(\infty) = 0 \tag{19}$$

The local skin friction coefficient defined as $C_f = \tau/(\frac{1}{2}\rho U_\infty^2)$ is now written in the following expression:

$$\tfrac{1}{2}C_fRe^{1/(n+1)} = \left[\frac{1}{(n+1)\xi}\right]^{n/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2n}\left(\frac{r}{L}\right)^n[f''(0)]^n \tag{20}$$

For forced convection in power-law flow over a nonisothermal rotating body, a second coordinate transformation is employed to facilitate the analysis of the thermal boundary layer. The new transformation variables are

$$X = [1 - (\xi_0/\xi)^c]^{1/3} \tag{21a}$$

$$\zeta = b(\xi)(\eta/X) \tag{21b}$$

$$\theta(X, \zeta) = [(T - T_\infty)/(T_s - T_\infty)] \tag{21c}$$

The transformation changes the expression of the energy equation into

$$\frac{\partial^2\theta}{\partial\zeta^2} + \frac{1}{(n+1)\xi}\beta\left[f + (n+1)\xi\frac{\partial f}{\partial\xi}\right]X\frac{\partial\theta}{\partial\zeta} + \frac{c}{3\xi b}\beta\frac{(1-X^3)}{X}f'\zeta\frac{\partial\theta}{\partial\zeta} - \frac{1}{b_2}\frac{\mathrm{d}b}{\mathrm{d}\xi}\beta X^2 f'\zeta\frac{\partial\theta}{\partial\zeta} - \frac{c}{3\xi b}\beta(1-X^2)f'\frac{\partial\theta}{\partial X} = 0 \tag{22}$$

where

$$\beta = \frac{nU_eL}{\alpha b}\left(\frac{r}{L}\right)^{n+1}\left(\frac{U_e}{U_\infty}\right)^{2n-3}\left[\frac{(n+1)\xi}{Re}\right]^{2/(n+1)} \tag{23}$$

with the boundary conditions

$$\theta(X, 0) = 1, \quad \theta(X, \infty) = 0 \tag{24}$$

In order to solve Eq. (22), the dimensionless temperature, stream function, and tangential velocity are expanded into

$$\theta(\xi, \eta) = \theta(X, \zeta) = \sum_{n=0}^{\infty}\theta_n(\zeta)X^n \tag{25}$$

$$f(\xi, \eta) = \sum_{m=2}^{\infty}a_m(\xi)\frac{\eta^m}{m!} \tag{26}$$

$$g(\xi, \eta) = \sum_{k=0}^{\infty}p_k(\xi)\frac{\eta^k}{k!} \tag{27}$$

The coefficients $a_m(\xi)$ and $p_k(\xi)$ can be determined by imposing the boundary conditions [Eqs. (9a) and (9b)], and then by substituting Eqs. (26) and (27) into the momentum equations [Eqs. (7a) and (7b)], respectively. They are

$$a_2 = f''(0), \quad a_3 = -\Lambda a_2^{1-n} - \tfrac{4}{9}W^2\Lambda$$
$$a_4 = (1-n)\Lambda^2 a_2^{1-2n} - \tfrac{8}{9}\Lambda W^2 a_2^{1-n}p_1 \tag{28}$$

$$p_0 = 1, \quad p_1 = g'(0), \quad p_2 = 0, \quad p_3 = 2\Lambda a_2(2Wp_1/3)^{1-n} \tag{29}$$

Now, substituting Eqs. (25–27) into Eq. (22), and arranging the terms with the same powers of X, a sequence of second-order differential equations is obtained. Further simplification of the equation can be achieved after substitution of the parameters c and b, defined as

$$c = \frac{3}{2(n+1)}, \quad b = \left[\frac{n}{6(n+1)\xi}\left(\frac{r}{L}\right)^{n-1}\left(\frac{U_e}{U_\infty}\right)^{2(n-1)} \times [(n+1)\xi]^{2/(n+1)}Pra_2\right]^{1/3} \tag{30}$$

The procedure yields the final form of the differential equations as

$$\frac{\partial^2\theta_0}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_0}{\partial\zeta} = 0 \tag{31}$$

with the boundary conditions

$$\theta_0(0) = 1, \quad \theta_0(\infty) = 0 \tag{32}$$

$$\frac{\partial^2\theta_1}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_1}{\partial\zeta} - 3\zeta\theta_1 = -\frac{3}{2b}\frac{a_3}{a_2}\zeta^3\frac{\partial\theta_0}{\partial\zeta} \tag{33}$$

with

$$\theta_1(0) = \theta_1(\infty) = 0 \tag{34}$$

$$\frac{\partial^2\theta_2}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_2}{\partial\zeta} - 6\zeta\theta_2 = \frac{3a_3}{2a_2b}\zeta^2\theta_1 - \frac{3a_3}{2a_2b}\zeta^3\frac{\partial\theta_1}{\partial\zeta} - \frac{a_4}{2a_2b^2}\zeta^4\frac{\partial\theta_0}{\partial\zeta} \tag{35}$$

with

$$\theta_2(0) = \theta_2(\infty) = 0 \tag{36}$$

$$\frac{\partial^2\theta_3}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_3}{\partial\zeta} - 9\zeta\theta_3 = \frac{3a_3}{a_2b}\left(2\zeta^2\theta_2 - \zeta^3\frac{\partial\theta_2}{\partial\zeta}\right) + \frac{a_4}{2a_2b^2}\left(\zeta^3\theta_1 - \zeta^4\frac{\partial\theta_1}{\partial\zeta}\right) + \left(3\zeta^2 - \frac{a_5}{8a_2b^3}\zeta^5 - \frac{9}{2(n+1)c}\zeta^2 - \frac{2\xi a_2'}{2a_2c}\zeta^2 + \frac{9\xi b'}{bc}\zeta^2\right)\frac{\partial\theta_0}{\partial\zeta} \tag{37}$$

with

$$\theta_3(0) = \theta_3(\infty) = 0 \tag{38}$$

The solution to Eqs. (31) and (33) can be obtained analytically as

$$\theta_0 = 1 - [\Gamma(\tfrac{1}{3}, \zeta^3)/\Gamma(\tfrac{1}{3})] \tag{39}$$

$$\theta_1 = M\bar{\theta}_1 = [M\zeta/5\Gamma(\tfrac{1}{3})][\Gamma(\tfrac{4}{3}) - \Gamma(\tfrac{4}{3}, \zeta^3)] \tag{40}$$

where $M = -3a_3/(2a_2b)$, and $\Gamma(n, \zeta^3)$ is an incomplete Gamma function defined as

$$\Gamma(n, \zeta^3) = \int_0^{\zeta^3} e^{-t}t^{n-1}\,\mathrm{d}t \tag{41}$$

With the known solution of θ_0 and θ_1, the function θ_2 is written as

$$\theta_2 = M^2\bar{\theta}_{2,1} + N\bar{\theta}_{2,2} \tag{42}$$

where $N = -a_4/(2a_2b^2)$.

Then Eq. (35) can be decomposed into

$$\frac{\partial^2\bar{\theta}_{2,1}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{2,1}}{\partial\zeta} - 6\zeta\bar{\theta}_{2,1} = -\frac{3\zeta^7e^{-\zeta^3}}{5\Gamma(\tfrac{1}{3})} \tag{43a}$$

$$\frac{\partial^2\bar{\theta}_{2,2}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{2,2}}{\partial\zeta} - 6\zeta\bar{\theta}_{2,2} = -\frac{3\zeta^4e^{-\zeta^3}}{\Gamma(\tfrac{1}{3})} \tag{43b}$$

The function θ_3 also is written as

$$\theta_3 = M^3\bar{\theta}_{3,1} + MN\bar{\theta}_{3,2} + P\bar{\theta}_{3,3} + Q\bar{\theta}_{3,4} \tag{44}$$

where $P = -a_5/(8a_2b^3)$ and

$$Q = 3\left[1 - \frac{3}{2(n+1)c} + \frac{3\xi}{bc}\frac{\mathrm{d}b}{\mathrm{d}\xi} - \frac{3\xi a_2'}{2ca_2}\right] \tag{45}$$

Thus, Eq. (37) becomes

$$\frac{\partial^2\bar{\theta}_{3,1}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{3,1}}{\partial\zeta} - 9\zeta\bar{\theta}_{3,1} = \zeta^3\bar{\theta}'_{2,1} - 2\zeta^2\bar{\theta}_{2,1} \quad (46a)$$

$$\frac{\partial^2\bar{\theta}_{3,2}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{3,2}}{\partial\zeta} - 9\zeta\bar{\theta}_{3,2} = \zeta^4\bar{\theta}'_1 - \zeta^3\bar{\theta}_1 + \zeta^3\bar{\theta}'_{2,2} - 2\zeta^2\bar{\theta}_{2,2} \quad (46b)$$

$$\frac{\partial^2\bar{\theta}_{3,3}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{3,3}}{\partial\zeta} - 9\zeta\bar{\theta}_{3,3} = \zeta^5\theta'_0 \quad (46c)$$

$$\frac{\partial^2\bar{\theta}_{3,4}}{\partial\zeta^2} + 3\zeta^2\frac{\partial\bar{\theta}_{3,4}}{\partial\zeta} - 9\zeta\bar{\theta}_{3,4} = \zeta^2\theta'_0 \quad (46d)$$

With the boundary conditions for Eqs. (43a–46d) being homogeneous, solutions to Eqs. (46c) and (46d) are also analytical, and they are

$$\bar{\theta}_{3,3} = \frac{e^{-\zeta^3}}{9\Gamma(\frac{1}{3})}(\tfrac{2}{3}\zeta + \zeta^4), \quad \bar{\theta}_{3,4} = \frac{\zeta e^{-\zeta^3}}{6\Gamma(\frac{1}{3})} \quad (47)$$

Analytical solutions to Eqs. (43a), (43b), (46a), and (46b) do not seem to exist. However, Eqs. (43a) and (43b) are identical to Eqs. (3.39) and (3.40) in Lee,[5] and Eqs. (46a) and (46b) are the same as Eqs. (45) and (46) in Kim et al.,[9] respectively. The numerical tabulation for each function in the entire range of ζ is given in the reference. The values of the first derivative of the function at the surface are quoted here for the calculation of the wall heat flux. They are

$$\begin{aligned}\bar{\theta}'_{2,1}(0) &= 0.0081748, & \bar{\theta}'_{2,2}(0) &= 0.040872\\ \bar{\theta}'_{3,1}(0) &= 0.0017205, & \bar{\theta}'_{3,2}(0) &= 0.012904\end{aligned} \quad (48)$$

If the Nusselt number if defined by

$$Nu = \frac{q_w L}{k(T_s - T_\infty)} \quad (49)$$

then

$$NuRe^{-1/(n+1)} = \left[\frac{(n^2+n)^{2/(n+1)}}{6(n+1)}\left(\frac{\xi}{n}\right)^{1-n/(n+1)} \times \left(\frac{U_e}{U_\infty}\right)^{2(n-1)}\left(\frac{r}{L}\right)^{n-1} Pra_2\right]^{1/3}\left[\frac{1}{(n+1)\xi}\right]^{1/(n+1)}\left(\frac{r}{L}\right) \times \left(\frac{U_e}{U_\infty}\right)\left(\frac{1}{X}\right)\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} \quad (50)$$

where

$$\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} = -\left[-1.1198 - \frac{a_3}{10a_2 b}X + (0.0081748M^2 + 0.040874N)X^2 + (0.0017205M^3 + 0.012904MN + 0.02765P - 0.062214Q)X^3 + \cdots\right] \quad (51)$$

The accuracy of Eq. (50) for a specified problem will depend upon the convergence of the series and the parameters such as the Prandtl number, rotating velocity, and the range of X. For large Pr, or in the region near the point of discontinuity in surface temperature, the convergence is fairly quick, so that only the first few terms of θ with one term solution of $f''_0(0)$ and $g'_0(0)$ for the velocity gradients are necessary for accurate results. In a region far downstream, slow convergence of the series may require a few more additional terms in θ, f, and g for high accuracy.

Forced Convection over a Rotating Sphere

For the flow over R, L is R. Therefore, the body contour $r(x)$ and the freestream velocity are given by

$$(r/R) = \sin(x/R) \quad (52)$$

$$(U_e/U_\infty) = \tfrac{3}{2}\sin(x/R) \quad (53)$$

For this case, the parameters of

$$\frac{(n+1)\xi}{r}\frac{dr}{d\xi}\left(\frac{r\Omega}{U_\infty}\right)^2, \quad \frac{2(n+1)}{r}\frac{dr}{d\xi}, \quad \left(\frac{r\Omega}{U_e}\right)^{1-n}$$

appearing in Eqs. (11–13) now become $(\frac{2}{3}W)^2\Lambda$, 2Λ, and $(\frac{2}{3}W)^{1-n}$, respectively.

The local friction coefficients in terms of $\frac{1}{2}C_f Re^{1/(n+1)}$ are computed using only the primary function $f''_0(0)$ in the series. The effects of n and the rotation parameters are shown in Fig. 2. It can be seen that, in the regions near the forward stagnation point and the separation point, the friction coefficient is larger in pseudoplastic fluids than both Newtonian and dilatant fluids. This result is attributed to the shear thinning effect of pseudoplastic fluids. The friction coefficient, in general, increases as the rotation parameter and x/R increase and reaches the maximum near $x/R = 1.0$. For a demonstration of the accuracy, the result is compared with that of Lee et al.[5] for the case of Newtonian flow, with $W = 1.5$, 3.0, and 4.74342 which correspond to their $B = 1.0$, 4.0, and 10, respectively. The numerical data obtained in this analysis are identical to the one-term solution of the above-mentioned reference, and shows good agreement with their three-term solution in almost the entire domain, except the region near the flow separation. For a further check of accuracy, the results shown in Fig. 2 are compared to the analysis provided by Kleinstreur and Wang.[10] The two results agree very well in the front portion of the sphere, but deviate as x/R increases. The discrepancy reaches approximately 10% at $x/R = 1.5$ for $W = 1.0$ (BP = 1.0 in Kleinstreuer and Wang[10]). If more terms in the series of the velocities are included, the discrepancy will diminish in the entire region of nonseparated flow.

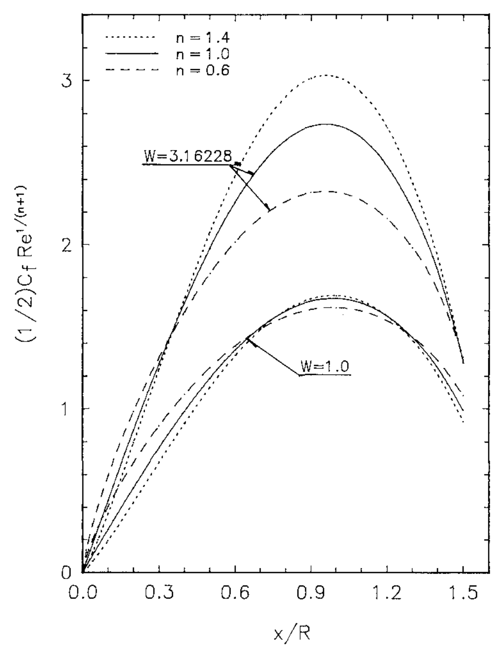

Fig. 2 Effect of power-law index and rotation parameter on friction coefficient.

The local heat transfer expressed in terms of $NuRe^{-1/(n+1)}$ is examined for various cases with different parameters. The values of $NuRe^{-1/(n+1)}$ are computed using Eq. (50) with the derivatives of a_2 and b with respect to ξ being omitted since the effect of the two terms on the final result is regarded as being small. The numerical data of $NuRe^{-1/(n+1)}$ for an isothermal rotating sphere are plotted in Fig. 3 for the purpose of demonstrating functional dependence of the heat transfer coefficient on the power-law index and the rotation parameter. The $NuRe^{-1/(n+1)}$ in pseudoplastic flows increases, as x/R increases, to reach the maximum at x/R being about 0.4, and then monotonically decreases. In Newtonian and dilatant flows, $NuRe^{-1/(n+1)}$ decreases as x/R increases. In general, $NuRe^{-1/(n+1)}$ increases as the Prandtl number and the rotation parameter increase in all Newtonian and non-Newtonian flows. However, energy transport in non-Newtonian flows shows a characteristic markedly different from that of Newtonian flows as x/R approaches zero. For dilatant flows, the value rapidly increases to infinity, but rapidly decreases to zero for pseudoplastic flows. As n approaches unity, the curves of $NuRe^{-1/(n+1)}$ near the forward stagnation point will become steeper and eventually converge to a nonzero constant at the stagnation point for $n = 1$ as shown in the figure. The opposite trend of psuedoplastic against dilatant flows is apparently caused by the formulation of shear stress for the power-law model, and subsequently, the variable $(\xi)^{(1-n)/(n+1)}$ in Eq. (50), as noted by Kleinstreuer and Wang.[10] However, it is difficult to clearly demonstrate the physical validity of the asympoteness of the curves in non-Newtonian power-law flows.

The accuracy of the present formula is checked by considering an isothermal sphere in a Newtonian flow. Since this condition makes the value of X constant at unity, and the convergence of the series is slow for larger values of X, the error resulting from carrying first few terms in the series is the largest, and this error sets the upper bound of error. The numerical comparison of $NuRe^{-1/2}$ by the current formula and the data in literature is made in Table 1. The present data agree very well with Lee et al.[5] in the region between $0 \leq x/R \leq 1.3743$. The data given by Jeng et al.,[6] with three-term velocity functions and the effect of the derivative of a_2 included, show some discrepancy. For non-Newtonian power-law flows, the curves of $NuRe^{-1/(n+1)}$ plotted in Fig. 3 show fairly good agreement with Kleinstreuer and Wang[10] for the entire region. The maximum discrepancy for dilatant fluids is within 3.5%, and that for pseudoplastic fluids is within 6%.

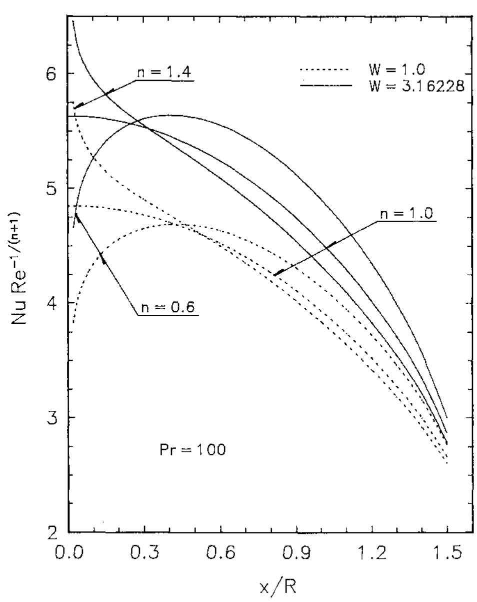

Fig. 3 Effect of power-law index and rotation parameter of $NuRe^{-1/(n+1)}$ for an isothermal sphere.

Table 1 Comparison of $NuRe^{-1/2}$ for an isothermal rotating sphere in Newtonian flow with $Pr = 1.0$

	$W = 1.5$			$W = 3.0$		
x/R	Present	Lee	Jeng	Present	Lee	Jeng
0.0000	0.9594	0.9588	0.9493	1.0258	1.0214	1.0007
0.4739	0.9212	0.9195	0.9116	0.9844	0.9789	0.9608
0.9507	0.8059	0.7998	0.7991	0.8592	0.8484	0.8422
1.2149	0.7074	0.6991	0.7063	0.7510	0.7328	0.7445
1.3743	0.6330	0.6171	0.6181	0.6673	0.6414	0.6544
1.4860	0.5697	——	——	0.5923	——	——

Table 2 Comparison of $NuRe^{-1/2}$ for a nonisothermal rotating sphere in Newtonian flow with $Pr = 1.0$

	$W = 1.5$		$W = 3.0$		$W = 4.5$	
x/R	Present	Jeng	Present	Jeng	Present	Jeng
			$x_0/R = 0.22$			
0.4739	0.9603	0.9524	1.0268	1.0049	1.1087	1.0765
0.9507	0.8106	0.8309	0.8642	0.8474	0.9296	0.9052
1.2149	0.7096	0.7086	0.7534	0.7480	0.8081	0.7997
1.3743	0.6345	0.6195	0.6689	0.6559	0.7138	0.7161
1.4860	0.5708	——	0.5935	——	0.6258	——
			$x_0/R = 0.9$			
0.9507	1.7840	1.6866	1.8335	1.8333	2.0513	2.0493
1.2149	0.8970	0.9011	0.9570	0.9582	1.0419	1.0429
1.3743	0.7453	0.7267	0.7876	0.7739	0.8480	0.8503
1.4860	0.6496	——	0.6761	——	0.7161	——

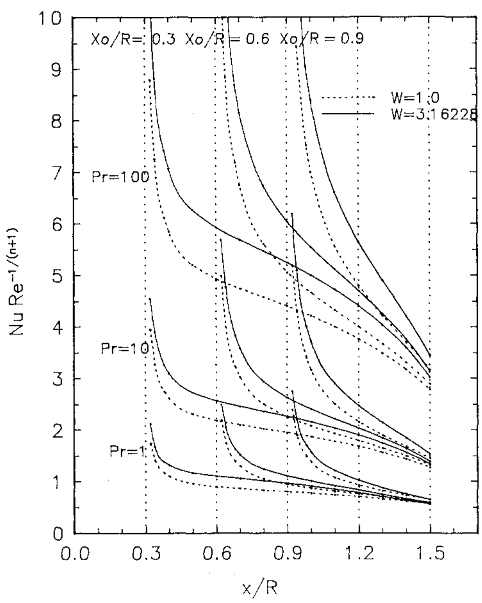

Fig. 4 Effect of Prandtl number and rotation parameter of $NuRe^{-1/(n+1)}$ for a nonisothermal sphere with $n = 0.6$.

Considering again that only single-term velocity functions are used, this is a surprising accuracy. With the inclusion of more terms in the series of the velocity functions, the accuracy will definitely improve.

For a nonisothermal rotating sphere, the same formula [Eq. (50)] is used to calculate the $NuRe^{-1/(n+1)}$. Two special cases of Newtonian flows with step discontinuity occurring at $x_0/R = 0.22$ and 0.9 are presented in Table 2, along with the currently available data in literature.[6] Two results exhibit generally good agreement with the maximum deviation reaching 3.2% at $W = 4.7434$ and $x/R = 0.4739$. The numerical data of $NuRe^{-1/(n+1)}$ for a rotating sphere with step discontinuity in surface temperature starting at $x_0/R = 0.3$, 0.6, and 0.9 are plotted in Fig. 4 for $W = 1.0$ and 3.16228 in pseudoplastic flows ($n = 0.6$). Figure 5 reveals $NuRe^{-1/(n+1)}$ at similar parameters, but in dilatant flows ($n = 1.4$). The figures show

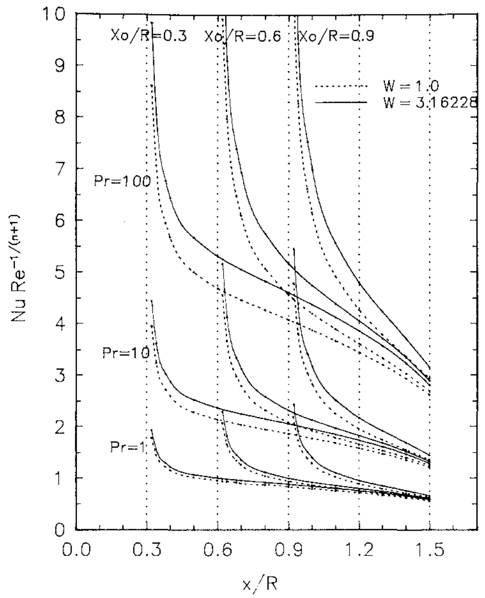

Fig. 5 Effect of Prandtl number and rotation parameter on $NuRe^{-1/(n+1)}$ for a nonisothermal sphere with n = 1.4.

that the rate of heat transfer increases as Prandtl number and the rotation parameter increase, but the effect of rotation parameter on the heat transfer is relatively smaller in dilatant flows than in pseudoplastic flows. No direct comparison of data is made to establish the accuracy of the result since it is believed that no previously published data is available.

Examples other than a rotating sphere are not included in this article. However, it should be noted that the present formula is readily applicable to other types of bodies, such as rotating disks and cones. For a rotating disk, no modification to the formula is necessary. If the freestream velocity over a rotating disk is given by $U_e/U_{\$} = 2x/\pi R$, Λ is equal to $(n + 1)/(3n + 1)$. For the given power-law index, Λ is a constant, so that only the primary functions for the axial and tangential velocities will be needed for friction and heat transfer coefficients. It will be interesting to apply the present formula to a more general body such as a rotating ellipsoid.

Conclusion

A simple, efficient, general solution method was developed to analyze the momentum and heat transfer in non-Newtonian power-law flows over a nonisothermal rotating body. The application of a series of coordinate transformations was successful in providing the friction and heat transfer coefficients in pseudoplastic, Newtonian, and dilatant flows over a rotating sphere with isothermal or nonuniform surface temperatures. The influence of power-law index, rotation parameter, and the Prandtl number on the transfer characteristics were shown. The rate of heat transfer generally increases as the rotation parameter and the Prandtl number increase. The role of rotation parameter in dilatant fluid flows is less significant than in pseudoplastic flows. The results of the present method show fairly good agreement with the published data available for special cases. It is certain that the accuracy of the analysis shall improve with the addition of higher-order terms in the series of the velocity and temperature functions.

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